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NUMERICAL MODELING OF SCALE EFFECTS FOR CIRCULAR CYLINDER IN THE THEORY OF THERMOELASTIC MATERIALS WITH VOIDS

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This article is relevant, as changes during the external loading may affect the stress state of the materials. The aim of this paper is to consider the numerical modeling of heating for circular cylinders in the frame of the theory of elastic materials with voids. A numerical solution is build using COMSOL Multiphysics software, where the implementation of the considered theory is realized based on the direct equation-definition approach. Constitutive relations were written in General form partial differential equation module. A matrix form of the equations for the two-dimensional case was used. Scale effects arising in considered problems are discussed. The classical solution is the particular case of the considered theory, when the coupling number tends to asero, i.e. when the micro-dilatation effects are small and do not affect the material's stress state. The limiting case in the case of the small value of the coupling number is the classical thermoelasticity solution.

Key words: Mindlin continuum, elasticity, Saint-Venant problem, kinematical variable, displacement boundary

INTRODUCTION

Elasticity with voids theory was presented in [1]. Linear approximation of this theory was presented in [2]. In the same paper, a solution was given to the problem of the pure beam bending. Independently, the same model was proposed in the paper [3]. It can be considered as a simplified model of the Mindlin continuum [4]. Only free deformations of volume (free dilatation) are allowed. The problem for the plate with a hole was solved in the paper [5]. The problem for the pressure vessel was solved in [6]. In paper [7] it was shown that within the framework of the elasticity with voids theory, there is no polynomial solution similar to the classical solution of the Saint-Venant problem.

Analytical and numerical solutions for various problems of beam bending were discussed in detail in [8]. It is shown in [8] that the stiffness of the beam with micro-dilatation in any bending tests should always be higher in comparison with the stiffness of such a beam in simple tension. In [9] example of metamaterial with behavior consistent with elasticity with voids, the theory is shown. Numerical simulation in COMSOL was recently implemented in the article [10]. In [11] formulation for thermoelasticity with voids was given.

Possible applications of the linear micro-dilatation elasticity are related to the description of the porous media, in which the volume fraction of pores changes during the external loading. These changes are not significant in absolute values may affect the stress state of the materials. Moreover, the extended variant of the micro-dilatation elasticity with irreversible changes of the pores volume fraction can be applied for the analysis of damage (voids) accumulation in materials. Presented in this paper numerical solution for the pressurized thick-walled cylinder can be used for the validation of the micro-dilatation effects been identified in the experimental test with micro-tubes made of porous materials, such as ceramics or some type of polymers. It is also possible to apply the micro-dilatation elasticity for the description of some type of lattice auxetic metamaterials.

MATERIALS AND METHODS

In elasticity with voids theory, there is an additional kinematical variable φ. The main relations of this theory were introduced in [2]. We write: Ω_v – void volume in current configuration, Ω_t – full-body volume, Ω_{vr} – void volume in reference configuration (Eqs. 1-2):

$$P = \frac{\Omega_v}{\Omega_t} \tag{1}$$

$$P_R = \frac{\Omega_{vr}}{\Omega_t} \tag{2}$$

We obtain (Eq. 3):

$$\phi = -(P - P_R) \tag{3}$$

The strain-displacement equations are the same as in the classical elasticity theory. u_i is a displacement vector,

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ε_{ij} is a strain tensor (Eq. 4):

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (4)$$

Equilibrium equations (when there are no body forces) (Eq. 5):

$$\sigma_{ij,j} = 0 \quad (5)$$

To balance the microvolume of the material, the equilibrium equations for nonclassical stresses are added (Eq. 6):

$$h_{i,i} + g = 0 \quad (6)$$

here h_i is the equilibrated stress vector, g is an intrinsic equilibrated body force [12].

The constitutive relations of the thermoelasticity with voids theory are (Eqs. 7-9):

$$\sigma_{ij} = \lambda\theta + 2\mu\varepsilon_{ij} + \beta\varphi\delta_{ij} - 3Ka_T\Delta T \quad (7)$$

$$h_i = \alpha\varphi_{,i} \quad (8)$$

$$g = -\xi\varphi - \beta(\theta - a_T\Delta T) \quad (9)$$

here σ_{ij} – stress tensor, $\theta=\varepsilon_{kk}\delta_{ij}$ – dilatation, λ,μ – Lame parameters, β,ξ,α – additional material constants, a_T – coefficient of linear expansion (Eq. 10):

$$K = \lambda + 2\mu/3 \quad (10)$$

Physical meaning of these constants was discussed in detail in [13] and [14-16]. It is convenient to introduce coupling number (Eq. 11) [17]:

$$N = \beta^2/(K\xi), N \in [0,1) \quad (11)$$

It was firstly introduced in [18] but in the other form (Eq. 12):

$$N = \beta^2/((\lambda + 2\mu)\xi) \quad (12)$$

COMSOL Multiphysics is used for numerical modelling. Constitutive relations are written in General form partial differential equation (PDE) module. The boundary value problem for the General form PDE module is written in divergent form. With boundary equations on Ω we get (Eq. 13):

$$\nabla\Gamma = p \text{ in } \Omega - n\cdot\Gamma = P \text{ on } \partial\Omega_p u = u^0 \text{ on } \partial\Omega_u \quad (13)$$

here Δ – Nabla operator, Γ – flow tensor (Cauchy stress tensor in classical elasticity), p is a bulk force vector, n is an outer normal vector on the part of the boundary for the region $\partial\Omega_p$, P is a vector of surface sources on the part of the boudary $\partial\Omega_u$, u is a kinematical variables vector, u^0 pre-defined values of kinematic variables on the boundary $\partial\Omega_u$.

COMSOL automatically converts the system of equations (13) in the weak formulation and solves it using the finite element method. It is convenient to use the matrix form of the equations for the two-dimensional case (Eqs. 14-17):

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \varphi \end{pmatrix} \quad (14)$$

$$\Gamma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \\ h_1 & h_2 \end{pmatrix} \quad (15)$$

$$p = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} \quad (16)$$

$$P = \begin{pmatrix} -P_1 \\ -P_2 \\ 0 \end{pmatrix} \quad (17)$$

In Figure 1 the geometry of the problem is shown. A disk with a hole is considered. Outer radius r_0, inner radius r_i. On the inner boundary $T_{r=ri}$=100° *K*. Initial disk temperature T_0=0° *K*.

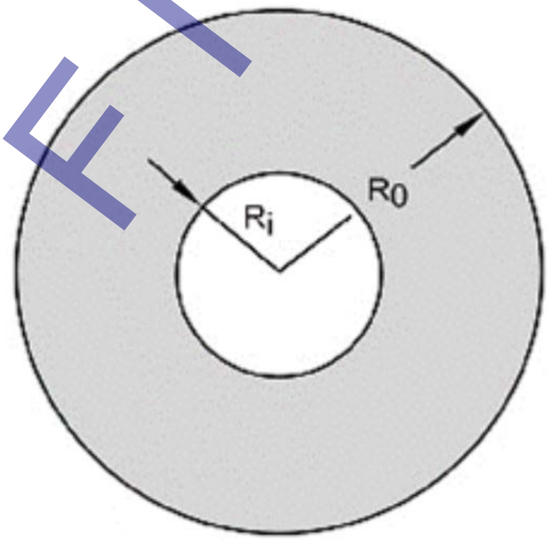

Figure 1: Cylinder geometry

RESULTS AND DISCUSSION

In classical elasticity theory when the stress state remaind the same for different values of r_0 (Eq. 18):

$$r_0/r_i = const \quad (18)$$

Classical thermoelasticity solution can be found in [10] (Eqs. 19-20):

$$\sigma_r = \frac{E\alpha_c T_i}{2\,log(r_0/r_i)}\left\{1 - log\left(\frac{r_0}{r}\right) - \frac{r_i^2}{r_0^2 - r_i^2}\left(1 - \frac{r_0^2}{r^2}\right) log\left(\frac{r_0}{r_i}\right)\right\} \quad (19)$$

$$\sigma_\theta = \frac{E\alpha_c T_i}{2\,log(r_0/r_i)}\left\{1 - log\left(\frac{r_0}{r}\right) - \frac{r_i^2}{r_0^2 - r_i^2}\left(1 + \frac{r_0^2}{r^2}\right) log\left(\frac{r_0}{r_i}\right)\right\} \quad (20)$$

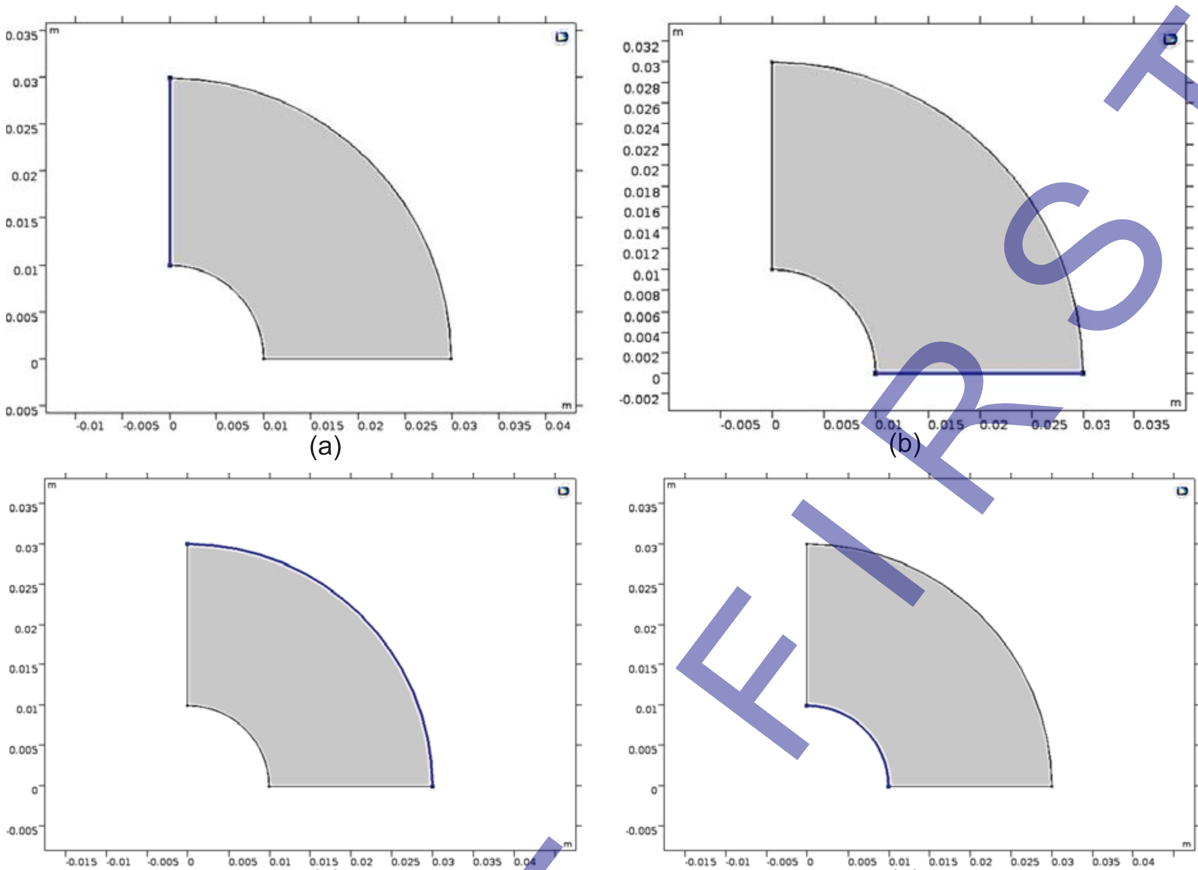

Figure 2: Displacement boundary conditions: a) u_x=0; b) u_y=0; temperature boundary conditions: c) $T_{r=ri}$=100, d) $T_{r=r0}$=0

In classical thermoelasticity when (18), we obtain similar stress states for different values of r_0. Let's compare the stress distribution for different values of *N* when (18). We change the outer radius r_0. In Fig. 3 and Fig. 4 distribution of σ_r and σ_θ for dimensionless radial distance for two different r_0 values. With the increase of r_0 stresses also rise. The black curve shows a classical solution for comparison.

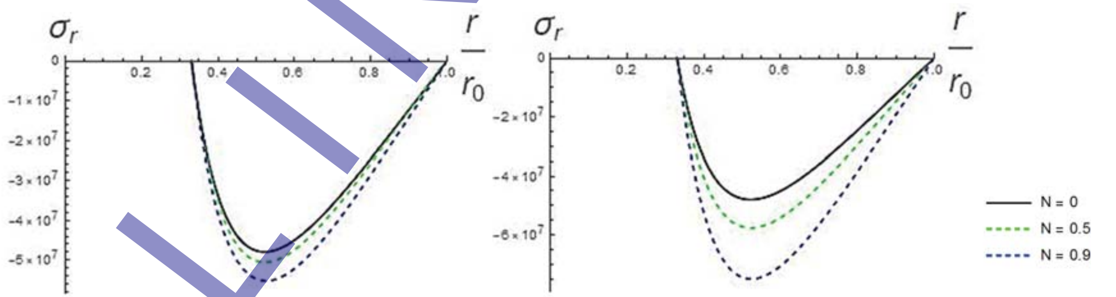

Figure 3: σ_r distribution for dimensionless radial distance; black – clsssical thermoelasticity, green – N=0.5, blue – N=0.9; a) Outer radius r_0=0.03, inner radius r_i=0.01; b) Outer radius r_0=0.3, inner radius r_i=0.1

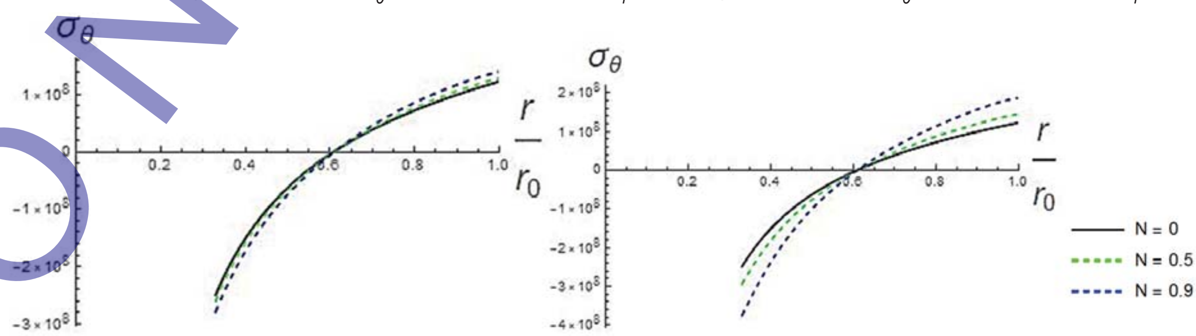

Figure 4: σ_θ distribution for dimensionless radial distance; black – classical thermoelasticity, green – N=0.5, blue – N=0.9; a) Outer radius r_0=0.03, inner radius r_i=0.01; b) Outer radius r_0=0.3, inner radius r_i=0.1

Consider the change in the stress-strain state with the increasing external radius r_0. For all cases (18). In Figure 5 and Figure 6 distribution of σ_r and σ_θ is shown for different r_0 when $N = const$. For comparison, the black curve shows the classic solution.

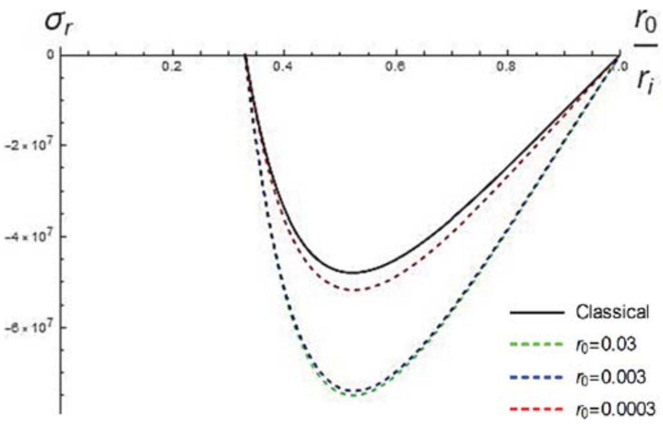

Figure 5: σ_θ distribution for dimensionless radial distance; For all cases: r_0/r_i=3, N = 0.9; black – classical thermoelasticity solution; thermoelasticity with voids solutions: green – r_0=0.3, blue – r_0=0.03, red – r_0=0.003

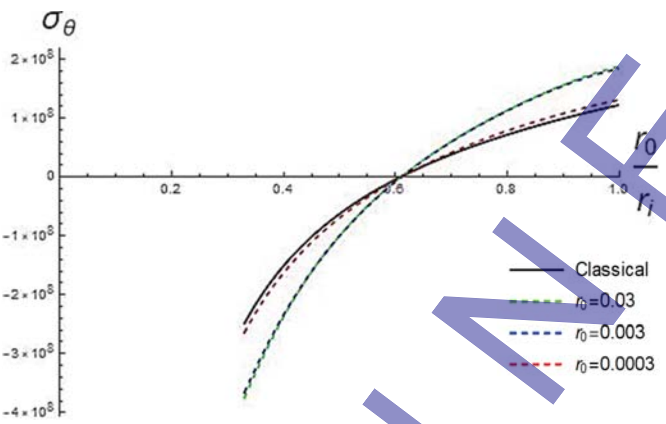

Figure 6: σ_θ distribution for dimensionless radial distance; for all cases: r_0/r_i=3, N = 0.9; black – classical thermoelasticity solution; thermoelasticity with voids solutions: green – r_0=0.3, blue – r_0=0.03, red – r_0=0.003

From the plots, you can see that for different values of r_0 when (18) we obtain different stress distributions. The maximum stresses increase with an increase of the external radius r_0, but reach the asymptote for sufficiently large values. Consider the function of microdilatation for different values. r_0 for (18). The distribution graphics are in Figure 7. Microdilatation ϕ changes strongly across the radius when r_0 is greater.

CONCLUSIONS

The presented solution describes the behavior of the structure with micro-dilatation, i.e. with valuable changes of porosity under external loading that affects the material stress state. Obtained dependences are not standard, in general, even for the non-classical theories, where the so-called micro-morphic effects usually vanish with an increase of the size of the relative structures with respect to the length scale parameter of the model.

The main results of this work are the following. The finite element solution for the circular cylinder in the theory of thermoelastic materials with voids is developed. It is shown that in elasticity with voids theory when: r_0/r_i=*const* stress-strain state differs for different values of r_0. Scale effects appear only in a limited range of values. For very large or very small sizes, the radial stresses will tend to constant values. The maximum stresses increase with an increase of the external radius but reach the asymptote for sufficiently large values. The classical solution is the particular case of the considered theory, when the coupling number tends to asero, i.e. when the micro-dilatation effects are small and do not affect the material's stress state. Presented numerical solution for the pressurized thick-walled cylinder can be used for the validation of the micro-dilatation effects been identified in experimental tests with micro-tubes made of porous materials.

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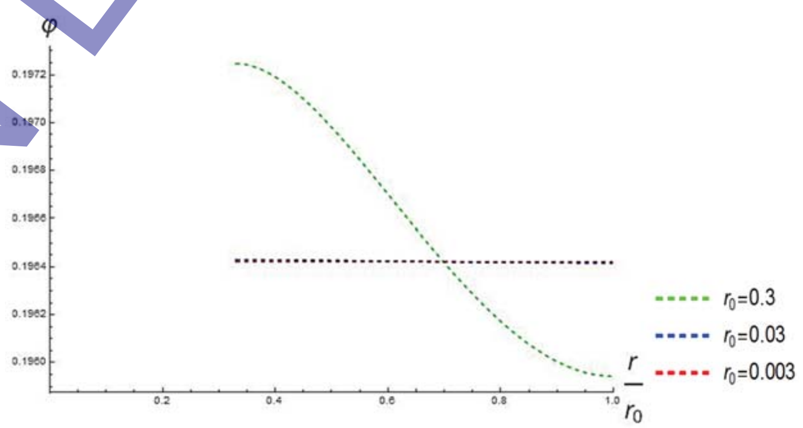

Figure 7: ϕ distribution for dimensionless radial distance; for all cases: r_0/r_i=3, N = 0.9; green – r_0=0.3, blue– r_0=0.03, red – r_0=0.003

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